INTRODUCTION

Tuberculosis (TB) is an airborne infectious disease, which may be transmitted among individuals who closely interact with one another. As an increasing threat to the global epidemic control, the historic misuse of antibiotic treatments has led to the emergence of multidrug-resistant tuberculosis (MDR-TB)**.** Many prominent management concerns still exist in the treatment of MDR-TB. Following the break of the Soviet Union, the exacerbation of MDR-TB worsened due to the rise in unemployment and crime rates (Olson et al., 2011). Due to the high poverty rate, the spread of MDR-TB increased in communities such as prison populations (Olson et al., 2011). The overall degradation of the health care system consequently allowed for an underestimation of the burden of MDR-TB.

As the causative agent of TB, *Mycobacterium tuberculosis* has re-emerged as a major public health threat (Mukherjee et al., 2004). The dynamics of this bacteria takes place over distinct temporal, spatial and organizational scales. Through the air, TB is mostly transmitted by persons coughing with pulmonary tuberculosis (Aparico and Castillo-Chavez, 2009). Typically, *M tuberculosis* attacks the lungs but it may also attack other parts of the body such as the kidneys and the spine and brain, which makes an improperly treated case fatal. The probability of transmission is generally low; even so, individuals at high risk of infection involve those who are exposed to infectious individuals for long periods of time (Aparico and Castillo-Chavez, 2009). Those who are symptomatic are often treated in a six month drug plan. The first two months consist of a daily intake of rifampin, isoniazid and pyrazinamide. The next four months consist of a daily intake of rifampin and isoniazid (Sandhu, G.K., 2011). Treatment of TB becomes difficult once a strain of *M tuberculosis* becomes resistant to isoniazid and rifampin.

Once a strain of *M tuberculosis* develops resistance to isoniazid and rifampicin, it is classified as MDR-TB (Sandhu, G.K., 2011). Without these two potent drugs, treatment of MDR-TB becomes difficult because second-line drugs must be used. Unlike the first-line agents, the second-line drugs are not as potent and as well tolerated by the body. Patients with MDR-TB have an advanced disease associated with thick-walled cavities and chronic lung lesions, making it difficult for antibiotics to penetrate (Mishra and Srivastava,. 2014). As a result, MDR-TB poses a substantial threat to household contacts and to MDR-TB control efforts (Mukherjee et al., 2004).  Even in highly developed health-care systems, outbreaks of MDR-TB have proven difficult to manage. Some cases are eventually controlled with a cost estimated at millions of dollars (Frieden et al., 1995).

Inconsistent drug supply and weak tuberculosis-control infrastructure can lead to a continuous cycle of inadequate treatment. This progresses transmission of drug resistant bacterial strains in resource-poor areas such as Russian prisons (Olson et al., 2011). This study will consider the effect of overpopulation in Russian prisons on the spread of MDR-TB. Using continuous mathematical models, a statistical analysis will be conducted to examine the complexity of the system. Using R, this study will reveal the dynamics of this continuous system of equations, furthering our understanding of the effect of overpopulation. Our results will reveal the importance of prison population controls, providing further guidance on the type of data required to predict the evolutionary and epidemiological dynamics of the *M tuberculosis* strain. By exploiting variables to control resistance this information can be used to reveal how to slow the spread of MDR-TB.

MODEL DERIVATION

Research models have been used to illustrate the effects of bacterial infectious disease.  Winetsky et al. created a research model which used key elements such as susceptible individuals (S), infected individuals (I) and processes such as treatment, transmission, death, and mutation from a normal strain of TB to MDR-TB (2012). Using this model, we examined the effect of overpopulation in prisons. This study will present the epidemiological basis of our analysis by modelling the rate of change of susceptible, TB infected and MDR-TB individuals in continuous time.

Our continuous model analyzed the effect of overcrowding in Russian prisons and its influence on the number of MDR-TB infected prisoners. We derived three equations measuring the population change over time of susceptible, infected, and MDR-TB infected prisoners (Table 1):

We included basic disease processes such as the transmission rates of TB (β), natural death rates (D), death from disease (d for TB and z for MDR-TB), and recovery rates *(r* for TB and *c* for MDR-TB*)* (Table 2). In addition, µ was used to describe the mutation rate of TB to MDR-TB. The prison population is determined by the incarceration rate (j) and release rate (f) which are independent of the prisoner’s infectious states and therefore, must be constant throughout the main equations of the model.

The transmission rates of TB (W) and MDR-TB (R) are functions of (N-K) which describe overpopulation in a prison, where N is the total number of prisoners and K is the carrying capacity of the prison. Based on the function β, it can be noted that transmission rate is directly proportional to the population dynamic, (N-K) with a slope of 𝛾. A positive difference in (N-K) correlates with a higher transmission rate; similarly, a negative difference in (N-K) correlates with a lower transmission rate. When the population is equal to the carrying capacity, the transmission rate is equivalent to the y-intercept, b.

Several assumptions were taken into account to simplify our mathematical model to focus on overpopulation. Our key assumptions are listed below:

1. All incoming prisoners are susceptible to TB.

2. The rate of mutation of TB (µ) is constant**.**

1. Transmission does not occur if an individual is already infected with TB or MDR-TB.

4. Incarceration rate and rate of release from jail are independent of the prison’s carrying capacity.

5. Prisoners with extensively drug-resistant Tuberculosis (XDR-TB) are treated as prisoners with MDR-TB.

6. Prisoners with TB and MDR-TB  become susceptible again immediately after recovery.

7. Other assumptions involve prison staff members, ventilation, nutrient conditions, treatment plans, and diagnostic methods.

Table 1

|  |  |
| --- | --- |
| Variables | Description |
| S | Number of susceptible prisoners |
|  | Number of prisoners infected with Tuberculosis |
|  | Number of prisoners infected with multiresistant Tuberculosis |

Table 2

|  |  |
| --- | --- |
| Parameters | Description |
|  | Transmission rate from susceptible prisoners to Tuberculosis infected prisoners; |
|  | Transmission rate from susceptible prisoners to MDR-TB; |
|  | Rate incarceration; |
| r | recovery rate for Tuberculosis infected prisoners; |
| c | recovery rate for MDR-TB infected prisoners; |
|  | Natural death rate; |
| d | death rate for TB infected prisoners; |
|  | death rate for MDR- TB infected prisoners; |
|  | rate of prisoners being released from prison |
| µ | Rate of mutation from TB to MDR TB |
| K | Prison capacity; [Number of prisoners] |
|  | total number of prisoners in a prison; [Number of prisoners] |
|  | Magnitude of overcrowding; [Number of prisoners] |
| b | rate of transmission when the total number of prisoner in a prison equal to the prison capacity |
|  | rate at which overcrowding affects transmission. |

ANALYSIS

The analysis of our model will be conducted using R, a software for computational statistics.

In our script, we will be assigning different parameters to (N-K). These parameters that describe the change in transmission rates of TB and MDR-TB will determine whether the prison is overpopulated, at capacity, or below capacity. We will then draw conclusions from these rates and determine how they affect equilibrium of the MDR-TB infected prisoners over time.

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